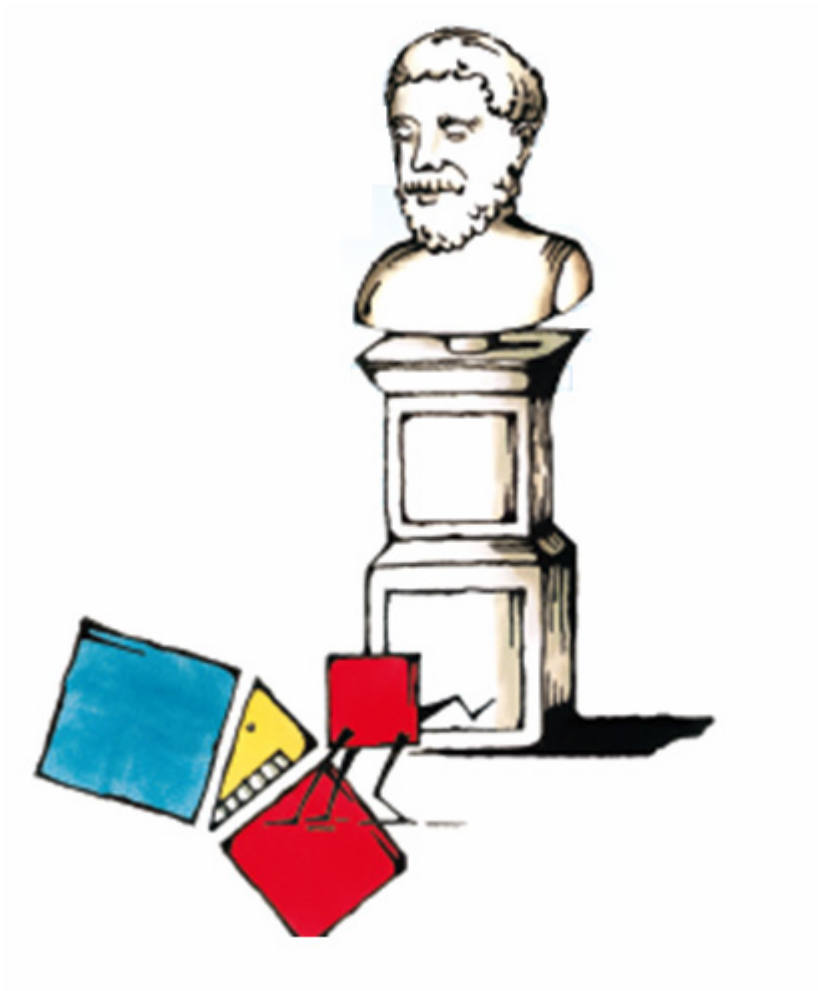


# PYTHAGORAS AND HIS THEOREM



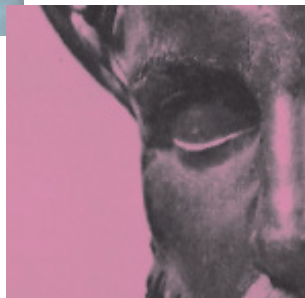


# 1. PYTHAGORAS



Scientist and shaman, legislator and oligarch, healer and philosopher (in fact, inventor of the term philosophy itself), magician and lecturer, liberator of cities and musician, father of Greek rationality and heir of the esoteric wisdom of the East, the founder of Mathematics is remembered mainly for a theorem discovered ten centuries before his birth and probably not proved until after his death. In short: Pythagoras.

The life of Pythagoras covers almost all of the sixth century before Christ. Born in Samos around 572 BC, he died in Metaponto around 494. The most important sources for his biography date from the early centuries AD. Porphyry and



Iamblichus, author of two biographies of the scientist, belong both to the third century of our era. An exposition of the Pythagorean philosophy can be found in Aristoteles (IV century BC) while his mathematics is described by Proclus (V century AD).

## 2. SAMOS, EGYPT, ITALY



### **Clemens Alexandrinus, Stromata I, 62**

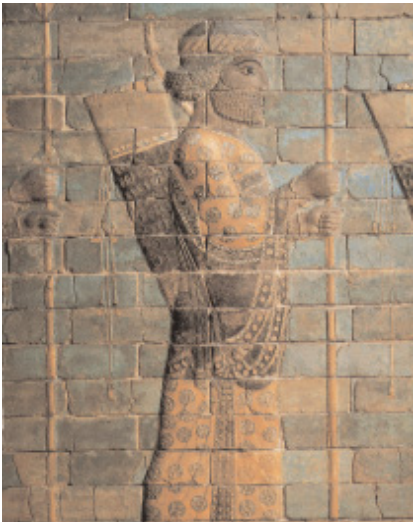
*According to Hippobotus, Pythagoras of Mnesarchus was from Samos. According to Aristoxenos, Aristarchus and Theopompus he was a Thyrranian. According to Neanthes a Syrian or a Thyrian. According to most writers, Pythagoras was thus of barbaric race.*

### **Porphyry, Life of Pythagoras 12**

*Thales led Pythagoras to sail to Egypt and meet with the priests of Memphis and Diospolis, because it was they who instructed him in those disciplines, for which he was called a sage by the people.*

### **Isocrates II, 28**

*Pythagoras of Samos, having gone to Egypt and become their disciple, first brought to Greece the study of every kind of philosophy .*



### **Theologumena Arithmetica 52**

*It is told that when Cambyses gained possession of Egypt, took prisoner Pythagoras, who lived there with the priests, and that Pythagoras, brought to Babylon, was there initiated to the mysteries. Cambyses lived precisely in the time of Polycrates, to escape whose tyranny Pythagoras had moved to Egypt.*

### **Porphyry, Life of Pythagoras 9**

*Aristoxenos says that at the age of forty, seeing that Polycrates' tyranny was too harsh for a free man to tolerate, Pythagoras left Samos and went to Italy.*

### **Porphyry, Life of Pythagoras 18**

*Dykearchus tells that, as soon as Pythagoras arrived in Italy and settled in Kroton, the Krotonese were so attracted to him (for he was a very remarkable man, and he had travelled much, and he had received from Fortune a splendid nature, a great and noble appearance, and much grace, and great decorum in speaking and behaving and everything else), that after he had captivated the Senate with many fine speeches, the magistrates charged him to speak to the young in words appropriate to their age. Thus his fame grew, and many became his companions, both citizen (not only men, but also women: and one of them, Theanos, became famous), and kings and lords of the surrounding region, inhabited by barbarians.*

# 3. THE TEACHING



## **Porphyry, Life of Pythagoras 6**

*About the object of his teaching, most historians say that he learned the so-called mathematical sciences from the Egyptians, the Chaldaeans and the Phoenicians, for already in ancient times the Egyptians had devoted themselves to the study of geometry, the Phoenicians to the study of arithmetics and logistic, and the Chaldaeans to the observation of the stars.*

## **Porphyry, Life of Pythagoras 36**

*Pythagoras exposed his teachings to his followers either in plain words or through symbols. Because his teaching was of two kinds, and those who attended were divided in Mathematicians and Acusmatics. The Mathematicians were those who knew the most important and deepest part of his doctrine, Acusmatics those who were summarily taught the rules, without any accurate explanations.*

## **Hyamblicus, Life of Pythagoras 82**

*The Acusmatics' philosophy consists of precepts: these are imparted without being shown why, or told the reason why one must act in a certain way. The Acusmatics also strive to conserve all his other sayings, and consider his words divine opinions, and of their own they say nothing and believe nothing should be said; indeed, they consider supremely wise those that have memorised more precepts and teachings*

*All these sayings are divided into three groups. Those of the first group answer the question: what is it? Those of the second to the question: what most of all? Those of the third to the question: what must one do or not do?*

*Examples of the first: What are the islands of the blessed? The sun and the moon. What is the oracle of Delphi? The tetrad, which is also the harmony of mermaids. [Examples of the second] Which is the wisest thing? The number, and secondly he who gave a name to things. Which is the most beautiful thing? Harmony. The most powerful thing? Intelligence. The best thing? Happiness. What is said with most truth? That men are evil.*

# 4. THE PYTHAGOREANS



## Hyamblicus, Life of Pythagoras 71-3

*He who had been thus examined by him would be left for three years in abandon, to ascertain how strong was his perseverance and his real desire to learn. Then he imposed upon his aspirants five years of silence, thus challenging their self-control. During this period, the goods of each of them - that is, his material possessions - were put in common, and entrusted to the appointed disciples. If they appeared worthy to be initiated to the doctrines, after five years of silence they became forever esoteric, they listened to Pythagoras inside the tent, and could even see him. Before, outside the tent, they could participate to his lessons only by listening, without ever seeing him.*

## Hyamblicus, Life of Pythagoras 96-98

*They took alone their morning walk in places where solitude and a proper calm reigned. After the morning walk they gathered preferably in temples, and they spent their time in teaching, learning and improving their character. After such occupation they took care of their bodies. For lunch they ate bread with honey or a honeycomb. During the whole day they never drank wine.*

*The afternoon was all dedicated to public administration business, to foreign politics and to the relationships with foreigners.*

*In the late afternoon they returned again to walk in groups of two or three, to recall to memory the knowledge learned and to practice in liberal studies. After their walk they took a bath and they went to the common banquet. After the banquet came the libations and then the reading. It was customary for the youngest to read, and for the oldest to decide what should be read, and how.*

## Diogenes Laertius, Life of the philosophers VIII, 15

*Those who went to hear him at night were no less than six hundred: and those who were allowed to see him wrote to their families that they had obtained an extraordinary favour. The Metapontines called his house the Temple of Demetra, and Museum the alley, as Favorinus writes.*

# 5. SEIZURE OF POWER AND REBELLION



## Hyamblicus, *De vita pythagorica* 33-34

*Tradition has it that during his stay in Italy and Sicily he liberated and set free the cities that he had found reciprocally subjected, after having filled them with the spirit of freedom through the followers that he had in each of them: Kroton, Sybaris, Catania, Reggio, Hymera, Agrigentum, Tauromenium and others. To these he gave the laws by Carondas of Catania and Zeleukos of Locris.*

## Hyamblicus, *De vita pythagorica* 254

*As long as Pythagoras was ready to converse with whomever got near him, he was liked in the city, but as soon as he started to talk only with his disciples, he lost their favour. Because, while they accepted that they were bested by a foreigner, they were*

*irritated by the locals that appeared privileged; and at the same time, they suspected that they were gathering to defeat them.*

*In addition, those youth came from the most illustrious and richest families, and in time they started not only to be leaders in their families, but at the same time they became the rulers of the city, having set up a big association (they were more than three hundred) although they were only a little part of the city.*

*The relatives, for their part, were irritated by the fact that the Pythagoreans shook others hands, but they didn't shake their relatives', apart from their parents; and they used substances in common, but excluded their relatives. They were the initiators of the rebellion, and the others followed promptly.*

*After this, having gathered a multitude, Cylon and Nynon, the first of a rich family, the second from the common people, began to accuse the Pythagoreans. And after Cylon had pronounced a long accusation speech, the other continued the attacks, boasting that he knew the secrets of the Pythagoreans.*

*In short, he said that their philosophy was nothing but a plot against the people, and he exhorted to not even let them speak: one shouldn't allow to speak those who had tried in every way to prevent others from speaking, but rather one should use force against them.*

# 6. ESCAPE FROM KROTON



## Hyamblicus, *De vita pythagorica* 248

*Cylon of Kroton was one of the first citizens by birth, by reputation and by wealth, but he was also harsh and violent and seditious and tyrannically minded. He had been taken by the desire to become part of the Pythagoreans' community, and he had asked Pythagoras himself, but he had been rebuffed. He had then, because of this, started a harsh war with his friends against Pythagoras and his friends. So violent was Cylon's and his companions' war, that it lasted until there were Pythagoreans left. Pythagoras was forced to flee to Metapontus, where, according to legend, he died.*

## Porphyrius, *Vita Pythagorae* 56

*Dycearchus and the most accurate of writers say that the plot took place while Pythagoras was in Kroton. And he writes that forty of his friends were attacked and captured in the house of one of them; the others, who were the majority, were killed here and there in the city, wherever they were found. And that Pythagoras, after the defeat of his followers, repaired first to the port of Caulonia, then he moved towards Locris, where, as soon as the news was received, some elders were sent to meet him at the region's borders. Having found him, they told him: "O Pythagoras, we know that you are an intelligent and wise man; but we are happy with our laws and we want them to remain as they are: so, if you need something, take it, but go somewhere else." Thus he was sent away from Locris. Hence, he passed on to Tarentum, where he had more or less the same luck that he had in Locris; then he moved on to Metapontus.*

## Diogenes Laertius, *Vitae philosophorum* VIII, 40

*Dycearchus says that Pythagoras, having repaired to Metapontus' temple of the Muses, died there after fasting for forty days.*





# 7. THE MYTH



## **Apollonius, Mirabilia 6**

*Pythagoras of Mnesarchus first devoted himself to mathematics and to numbers, then started to perform miracles, like Pherecides had. One day in Metapontum, as a ship was about to enter the harbour, [...] Pythagoras appeared and said: "This ship will bring you a dead man." Another time in Caulonia, Aristotle reports, he predicted the arrival of the white she-bear. And the same Aristotle says of him many other things: among others this: that responding with a bite to the bite of a deadly serpent, he killed it. And that he had predicted the sedition against the Pythagoreans.*

*Once he also appeared, on the same day and at the same time, both in Kroton and in Metapontus. Aristotle tells also that once, in a theater, he stood up and showed to the spectators that his thigh was made of gold.*

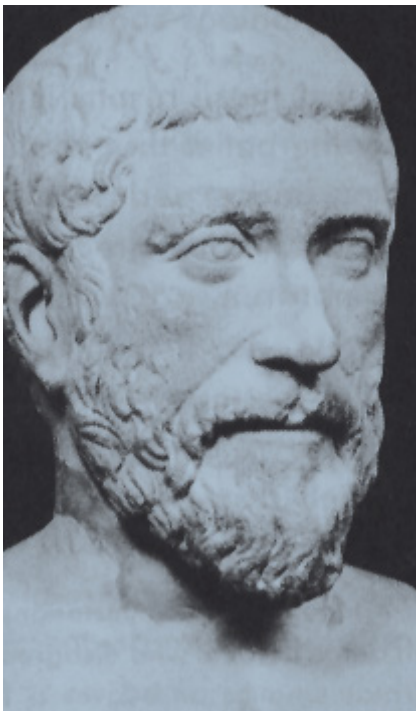
## **Helianus, Varia historia IV, 17**

*Pythagoras taught men that he had been born out of a better seed than that which begets*

*mortals. They tell that he was seen in Metapontus and in Kroton on the same day and at the same time. And in Olympia, he showed his thigh was made of gold. And he recalled that Myllia of Kroton had once been Mydas of Gordia, the Phrygian; and he stroked the white eagle, which did not flee from him.*

## **Hyamblicus, De vita pythagorica 31**

*Aristotle tells that one of the greatest secrets that the Pythagoreans keep is this distinction: living beings endowed with reason are divided into gods, humans, and beings like Pythagoras.*



# 8. THE BIRTH OF PHILOSOPHY



**Diogenes Laertius, Vitae philosophorum I, 12**

*Pythagoras first used the term philosophy and first called himself philosopher: nobody is indeed wise, except the divinity.*

**Cicero, Tusculanae Disputationes V, 3,8**

*There are some rare people that neglect completely everything else and attentively study nature. These are called lovers of knowledge, that is philosophers, and as in the market the noblest attitude is to be a spectator without seeking any advantage, so in life the study and the knowledge of things is by far superior to any activity. Indeed Pythagoras was not only the inventor of the name, but he developed the activity itself.*

**Porphyrius, Vita Pythagorae 18.**

*What he said to his companions, nobody can say for sure, because they kept a great secret about this. But his most known opinions are these: he said that the soul is immortal, and that it passed to animate beings of another species, then that what has been repeats itself in regular intervals, and there is nothing that is truly new; finally that all animate beings must be considered as belonging to the same race.*

**Diogenes Laertius, Vitae philosophorum VIII, 24**

*The beginning of every thing is the monad, and from the monad the infinite diad is born, subjacent as matter to the monad which is the cause: from the monad and the infinite diad come numbers, and from numbers points, and from these lines, and from these the flat figures, and from these solids, and from these the perceivable bodies, whose elements are four: fire, water, earth, air, which mutate and move through the everything.*

**Diogenes Laertius, Vitae philosophorum VIII, 15**

*Before Philolaus it was not possible to know the thought of Pythagoras: it was Philolaus who divulged the three famous books, which Plato had bought for the price of one hundred minae.*

**Aetius, I 21, 1**

*Pythagoras said that time is the sphere of that which envelopes things.*

# 9. COSMOLOGY



## **Aristotle, De coelo B 13 293 a 18**

*For most, Earth is in the centre. The Italics, called Pythagoreans, say the opposite: they say that in the centre there is fire, that the Earth is a star and that, rotating around its central part, it creates day and night. Then, opposite to that, they say there is a second Earth, which they call anti-Earth: and they say this not seeking the causes and the reasons in the phenomenon, but forcing the significance of phenomenon and trying to accord them with some preconcept reasons and opinions of theirs. And many others, provided they didn't try to find a certitude in the appraisal of phenomenon rather than in talks, would agree with them in denying that Earth is in the middle. Because they say that the most honourable place is due to the most honourable things, and that fire is worth more than Earth, and the extremes more than the parts comprised between them: and the circumference and the centre are extremes.*

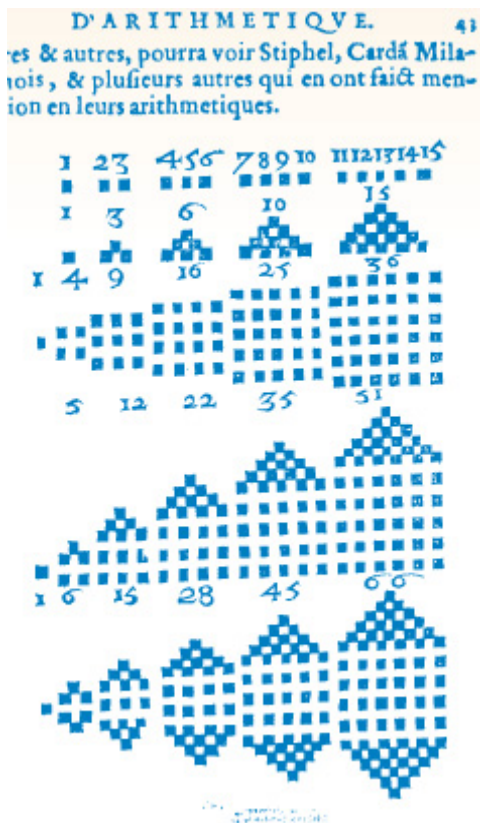
## **Aristotle, Meteorologica A 8. 345 a 13**

*Of the so-called Pythagoreans, some say that the milky way is a road, and some add that it is the road once followed by one of the stars, fallen in the ruin which is said to have happened in the days of Phaeton; others that it is the road followed by the Sun itself in its circular motion, and which was burned or had something happen to it while the Sun followed it.*

## **Diogenes Laertius, IX 23**

*Favorinus writes that Parmenides appears to have been the first to say that Vesper and Luciferus are the same star: others say that it was Pythagoras.*

# 10. NUMBERS



## Stobaeus, Eclogae physicae I pr. 6

*It appears that Pythagoras most of all appreciated the study of numbers, and that, emancipating it from the service of the merchants, he made it progress, and that he compared everything to numbers. Because the number contains all other things, and all numbers are in relation to one another.*

## Aristotle, Metaphysica A 5. 985 b 23

*The first to devote themselves to mathematics and to make them progress were the so-called Pythagoreans. They, devoted to this study, believed that the principles of mathematics were also the principles of all things that be. Now, since the principles of mathematics are numbers, and they thought they found in numbers, more than in fire and earth and water, similarities with things that are and that become (they judged, for example, that justice was a particular property of numbers, the soul and mind another, opportunity another, and similarly, so to say, anything else), and since furthermore they saw expressed by numbers the properties and the ratios of harmony, since finally everything in nature appeared to them to be similar to*

*numbers, and numbers appeared to be first among all there is in nature, they thought that the elements of numbers were the elements of all that there is, and that the whole world was harmony and number. And all the properties they could find in numbers and in musical chords, corresponding to properties and parts of the sky, and in general to the whole cosmic order, they gathered and adapted to it. And if something was missing, they made an effort to introduce it, so that their tractation be complete. To clarify with an example: since ten seems to be a perfect number and to contain in itself the whole nature of numbers, they said that the bodies that move in the sky are also ten: and since one can only see nine, they added as tenth the anti-Earth.*

## Aetius I 3, 8

*He said that the nature of numbers can be found in the decade: indeed all Greeks and all barbarians count to ten, then, once they have reached it, go back to the one. And the power of ten, he said, is in four and in the tetrad. And this is the cause: if one starts from one and adds numbers till one reaches four, one makes number ten. In other terms, if one takes one and adds two, three and then four, one gets ten. Thus the number is by units ten, but four by power.*

*And our soul, he said, is composed of the tetrad, being intelligence, knowledge, opinion, perception, whence comes to us every art and every knowledge and the ability to reason.*

# 11. HARMONIES



## Theon of Smyrne, 59.4

*These harmonies, some thought to originate from weights, some from sizes, some from vibrations and numbers, some from vases and sizes. Lasus from Hermion (and those of the school of Hippasus of Metapontus), according to tradition, judging that the speed and slowness of the vibrations whence harmonies arise could be expressed by the series of numeric ratios, obtained these harmonies using vases. He took some vases of the same size, and, while he left one empty, he filled the other by half with water; then he struck both and he obtained the octave. Then, again leaving one of the vases empty, he filled the other up to one quarter, then again he struck them both and obtained the fourth; he obtained the fifth when he filled up the vase one-third of the way. The*

*relationship between the emptiness of a vase and that of the other was then 2 to 1 for the octave, 3 to 2 in the fifth, and 4 to 3 for the fourth.*

## Scholia in Platonis Phaedrum 108 D

*Because, as a Hyppasus had built four bronze discs with the same diameter but different thickness (as the thickness of the first one was  $\frac{4}{3}$  of the second one's,  $\frac{3}{2}$  of the third's and twice the fourth's), and since those discs produced harmonic sounds when struck, Glaucus, it is said, having observed the harmony of sounds produced by the discs, firstly used them to play music. It is because of this activity that it is still called "the art of Glaucus".*



# 12. GEOMETRY



## Proclus, In Euclidem I 47

*In right-angled triangles, the square of the hypotenuse is equivalent to the sum of the squares of the sides that form the right angle. If one listens to those who concern themselves with antiquities, who attribute this theorem to Pythagoras, one will also find that some say he sacrificed an ox for this discovery.*

## Proclus, In Euclidem I 32

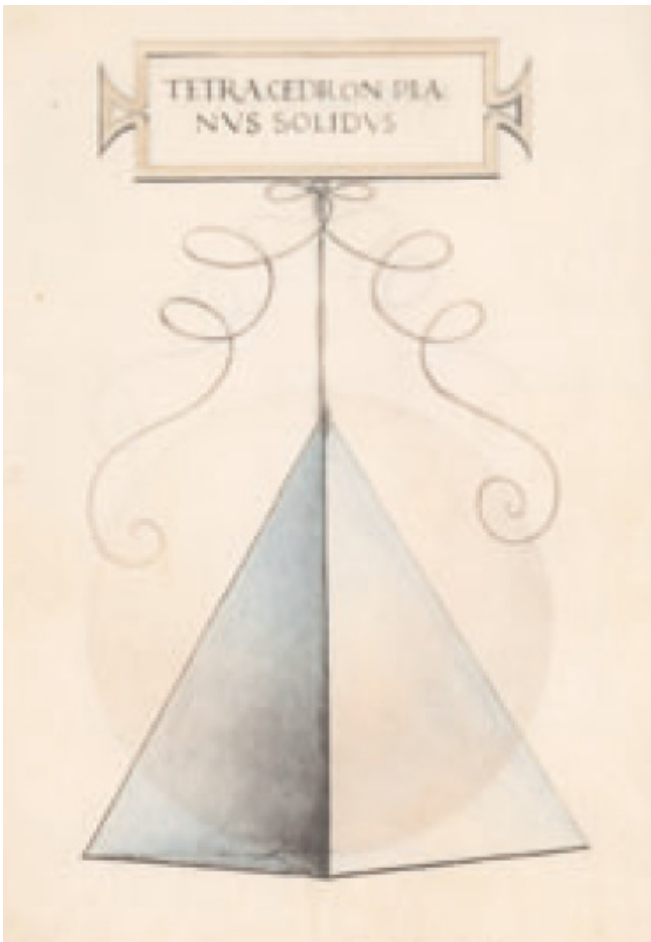
*If one prolongs one of the sides of any triangle, the external angle is equal to the sum of the two opposed internal angles, and the sum of the three internal angles of the triangle is equal to two right angles. The peripatetic Eudemus attributes the discovery of this theorem to the Pythagoreans.*

## Proclus, In Euclidem I 44

*Apply to a given segment in an angle equal to a given angle a parallelogram equivalent to a given triangle. Eudemus says that these discoveries: the application of figures and the hyperbole and the ellipse, are ancient, and that they are due to the muse of the Pythagoreans.*



# 13. REGULAR SOLIDS



**Proclus, In Euclidem 65, 11**

*Pythagoras devoted himself to the study of geometry, and he gave it the form of a liberal education, seeking its prime principles and investigating its theorems conceptually and theoretically: he was the first to deal with the irrational and to find the structure of cosmic figures.*

**Hyamblicus, De communi Mathematica scientia 25; De vita pythagorica, 88**

*They say that Hippasus was a Pythagorean, that he divulged and described the sphere formed by the twelve pentagons, that he died because of this impiety in a shipwreck, and that he was known as being the author of the discovery, while all the discoveries were by "that man", as they call Pythagoras without naming him.*



# 14. IRRATIONALS



## Hyamblicus, De vita pythagorica 246-247

*They say that the man who first divulged the nature of commensurability and incommensurability to men who were not worthy of being made part of this knowledge, became so much hated by the other Pythagoreans, that not only they cast him out of the community; they built a shrine for him as if he were dead, he who had once been their friend. Others add that even the god became angry with him who had divulged Pythagoras' doctrine; that he who showed how the icosagon (that is the dodecahedron, one of the five solid figures) can be inscribed within a sphere, died at sea like an evil man. Others still say that the same misfortune happened on him who spoke to others of irrational numbers and incommensurability.*

